## Nonlinear interactions between drift waves and zonal flows

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**Abstract.** Phase coherent interactions between drift waves and zonal flows are considered. For this purpose, mode coupling equations are derived by using a two-fluid model and the guiding center drifts. The equations are then Fourier analyzed to deduce the nonlinear dispersion relations. The latter depict the excitation of zonal flows due to the ponderomotive forces of drift waves. The flute-like zonal flows with insignificant density fluctuations have faster growth rates than those which have a finite wavelength along the magnetic field direction. The relevance of our investigation to drift wave driven zonal flows in computer simulations and laboratory plasmas is discussed.

**PACS.** 52.35.Mw Nonlinear phenomena: waves, wave propagation, and other interactions (including parametric effects, mode coupling, ponderomotive effects, etc.) - 52.35.Kt Drift waves - 52.35.Ra Plasma turbulence

### **1** Introduction

Recently, there has been a great deal of interest [1-8] in studying the excitation of zonal flows and streamers [9,10] by low-frequency drift waves and kinetic drift-Alfvén waves in a nonuniform magnetoplasma. Zonal flows and streamers are associated with zero-frequency convective cells [11,12]. The latter are either azimuthally symmetric two-dimensional [11], or pseudo-three-dimensional [12] long wavelength (in comparison with the ion gyroradius  $\rho_i$ ) electrostatic perturbations. The two-dimensional convective cells of Okuda and Dawson [11] have  $e\psi/T_{\rm e} \gg$  $n_{j1}/n_0$ , while the pseudo-three-dimensional convective cells of Hasegawa and Mima [12] have  $e\psi/T_{\rm e} \sim n_{j1}/n_0$ , where e is the magnitude of the electron charge,  $\psi$  is the electric potential of the convective cells/zonal flows,  $T_{\rm e}$ is the electron temperature, and  $n_{i1} \ll n_0$  is a small perturbation in the equilibrium density  $(n_0)$  of the particle species j (j equals e for the electrons and i for ions). The parallel (to  $\hat{\mathbf{z}}B_0$ , where  $\hat{\mathbf{z}}$  is the unit vector along the z-axis and  $B_0$  is the strength of the external magnetic field) wavelength of the pseudo-three-dimensional convective cells is supposed to be much smaller than the collisional electron mean free path. The convective cells are damped due to the ion gyroviscosity. The damping rates of the two and three-dimensional convective cells are, respectively,

$$\operatorname{Im} \omega = \frac{0.3k_{\perp}^2 \rho_{\rm i}^2 \nu_{\rm i}}{(1 + \omega_{\rm ci}^2 / \omega_{\rm pi}^2)}$$

and

Im 
$$\omega = \frac{0.3k_{\perp}^4 \rho_{\rm s}^2 \rho_{\rm i}^2 \nu_{\rm i}}{(1+k_{\perp}^2 \rho_{\rm s}^2)},$$

where  $\mathbf{k}_{\perp}$  is the perpendicular (to  $\hat{\mathbf{z}}$ ) component of the wave vector  $\mathbf{k}$ ,  $\nu_{i}$  is the ion-ion collision frequency,  $\omega_{pi}$ and  $\omega_{ci}$  are the ion plasma and ion gyrofrequencies, respectively,  $\rho_{s} = c_{s}/\omega_{ci}$  is the ion gyroradius at the electron temperature, and  $c_{s}$  is the ion sound speed. Thus, in a dense plasma ( $\omega_{pi} \gg \omega_{ci}$ ) long wavelength (in comparison with  $\rho_{s}$ ) convective cells have a very long lifetime, and they therefore cause cross-field particle transport even in a thermal equilibrium plasma.

However, the presence of low-frequency (in comparison with  $\omega_{ci}$ ) drift waves can excite zonal flows due to parametric interactions [13]. This happens because the drift wave frequency

$$\omega = \frac{k_y U_*}{(1 + k_\perp^2 \rho_s^2)}$$

peaks around  $k_y \rho_s \sim 1$ , and there exists a possibility of producing a zero-frequency ponderomotive force due to the beating of two drift waves which have identical frequencies (but different perpendicular wavelengths) in the short wavelength part of the drift wave spectrum; here  $U_* = c_s \rho_s / L_n$  is the electron diamagnetic speed and  $L_n = (\partial \ln n_0 / \partial x)^{-1}$  is the scale length of the density gradient which is supposed to be along the x-axis. The ponderomotive force, in turn, reinforces the zonal flows. It is widely believed that nonlinearly excited zonal flows play a very important role in controlling plasma transports in turbulent environments.

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In this paper, we re-examine the nonlinear excitation of zonal flows by coherent drift waves [13] in order to give some guidance to experimentalists and non-specialists in plasma physics. By using the two-fluid model and the drift approximations for the electrons and ions, we derive the mode coupling equations for drift waves and zonal flows accounting for the nonlinear interactions between them. A normal mode analysis is carried out to derive compact dispersion relations. The latter show that the ponderomotive force of coherent drift waves excite efficiently zonal flows which have insignificant density fluctuations. The results are in accord with those observed in computer simulations [14] and laboratory experiments [15, 16].

### 2 Governing equations

We consider a nonuniform magnetoplasma containing large amplitude electrostatic drift waves which are nonlinearly interacting with zonal flows. The perpendicular components of the electron and ion fluid velocities in the presence of nonlinearly coupled low-frequency (in comparison with  $\omega_{\rm ci} = eB_0/m_{\rm i}c$ , where  $m_{\rm i}$  is the ion mass and c is the speed of light in vacuum) and long wavelength (in comparison with the ion gyroradius  $\rho_{\rm i} = v_{\rm ti}/\omega_{\rm ci}$ , where  $v_{\rm ti}$  is the ion thermal speed) drift waves and zonal flows are, respectively,

$$\mathbf{v}_{e\perp}^{d} \approx \mathbf{v}_{E} + \mathbf{v}_{De},\tag{1}$$

and

$$\mathbf{v}_{i\perp}^{d} \approx \mathbf{v}_{E} + \mathbf{v}_{Di} + \mathbf{v}_{\pi i} - \frac{c}{B_{0}\omega_{ci}} \left(\partial_{t} + \mathbf{v}_{i\perp}^{d} \cdot \nabla - \mu_{i}\nabla_{\perp}^{2}\right) \nabla_{\perp}\phi$$
$$- \frac{c^{2}}{B_{0}^{2}\omega_{ci}} \left[ (\hat{\mathbf{z}} \times \nabla\phi \cdot \nabla)\nabla_{\perp}\psi + (\hat{\mathbf{z}} \times \nabla\psi \cdot \nabla)\nabla_{\perp}\phi \right], \quad (2)$$

$$\mathbf{v}_{\mathrm{e}\perp}^{\mathrm{z}} \approx \frac{c}{B_0} \hat{\mathbf{z}} \times \nabla \psi \tag{3}$$

and

$$\mathbf{v}_{i\perp}^{z} \approx \frac{c}{B_{0}} \hat{\mathbf{z}} \times \nabla \psi - \frac{c}{B_{0}\omega_{ci}} \times \left[ (\partial_{t} - \mu_{i}\nabla_{\perp}^{2})\nabla_{\perp}\psi + \frac{c}{B_{0}} \langle (\hat{\mathbf{z}} \times \nabla \phi \cdot \nabla)\nabla_{\perp}\phi \rangle \right], \quad (4)$$

where  $\phi$  and  $\psi$  are the electrostatic potentials of the drift waves and zonal flows, respectively,  $\mathbf{v}_{\rm E} = (c/B_0)\hat{\mathbf{z}} \times \nabla \phi$ is the  $\mathbf{E} \times \mathbf{B}_0$  velocity,  $\mathbf{v}_{\rm Dj} = (cT_j/q_jB_0n_0)\hat{\mathbf{z}} \times \nabla n_{j1}$ is the diamagnetic drift velocity of the particle specie j $(q_{\rm e} = -e \text{ and } q_{\rm i} = e)$ ,  $T_j$  is the temperature,  $\mathbf{v}_{\pi \rm i} = (\hat{\mathbf{z}} \times \nabla \cdot \mathbf{\Pi}_{\rm i})/en_0B_0$  is the drift velocity involving the collisionless gyroviscosity tensor [17]  $\mathbf{\Pi}_{\rm i}$ , and  $\mu_{\rm i} = (3/10)\nu_{\rm i}\rho_{\rm i}^2$ represents the collisional ion gyroviscosity [11]. The superscripts d and z represent quantities associated with the drift waves and zonal flows, respectively. The angular brackets denote averaging over one period of the drift waves. Substituting (1) into the electron continuity equation and eliminating the parallel component of the electron fluid velocity (with  $\nu_{\rm e} v_{\rm ez} \gg \partial_t v_{\rm ez}$ ) by means of the equation

$$n_{\rm e}\nu_{\rm e}v_{\rm ez}^{\rm d} \approx \partial_z \left(e\phi - \frac{T_{\rm e}}{n_0}n_{\rm e1}^{\rm d}\right)$$
 (5)

we obtain for  $(v_{\rm te}^2/\nu_{\rm e})\partial_z^2\gg\omega\sim k_y U_*$ 

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$$n_{\rm e1}^{\rm d} \approx n_0 e \phi / T_{\rm e},$$
 (6)

which is the Boltzmann electron density perturbation. Here  $m_{\rm e}$ ,  $\nu_{\rm e}$  and  $v_{\rm te}$  are the electron mass, the electron collision frequency, and the electron thermal speed, respectively.

The equation for the drift waves in a dense plasma (with  $\omega_{\rm pi} \gg \omega_{\rm ci}$ ) can be deduced from the ion continuity equation by inserting  $n_{\rm i} = n_0 + n_{\rm i1}^{\rm d} + n_{\rm i1}^{\rm z}$ , where  $n_{\rm i1} \ll n_0$ , and using equations (2, 4, 6) with  $n_{\rm i1}^{\rm d} \approx n_{\rm e1}^{\rm d}$ . The result for  $T_{\rm e} \gg T_{\rm i}$  is

$$\partial_t \phi - \rho_s^2 \nabla_\perp^2 \partial_t \phi - U_* \partial_y \phi + \frac{c}{B_0} \hat{\mathbf{z}} \times \nabla \psi \cdot \nabla \left( \phi - \rho_s^2 \nabla_\perp^2 \phi \right) = 0 \quad (7)$$

for zonal flows with  $e\psi/T_{\rm e} \ll n_{\rm i1}^{\rm z}/n_0$  and zonal wavelength much larger than  $\rho_{\rm s}$ . We have here assumed that the drift wave frequency is much larger than the damping rate  $\nu_{\rm i}\rho_{\rm i}^2k_{\perp}^2$ . On the other hand, the drift wave equation in the presence of the Hasegawa-Mima convective cells/zonal flows is of the form

$$\partial_t \phi - \rho_{\rm s}^2 \nabla_\perp^2 \partial_t \phi - U_* \partial_y \phi - \frac{c}{B_0} \rho_{\rm s}^2 \\ \times \left[ (\hat{\mathbf{z}} \times \nabla \psi \cdot \nabla) \nabla_\perp^2 \phi + (\hat{\mathbf{z}} \times \nabla \phi \cdot \nabla) \nabla_\perp^2 \psi \right] = 0, \quad (8)$$

with the Hasegawa-Mima scaling

$$n_{\rm e1}^{\rm z} = n_0 \frac{e\psi}{T_{\rm e}},\tag{9}$$

and  $n_{i1} = n_{e1}$ .

Next, we obtain the equation for azimuthally symmetric zonal flows by inserting (3) and (4) into the modified charge current density equation. We have

$$\left(\partial_t - \mu_{\rm i} \nabla_{\perp}^2\right) \nabla_{\perp}^2 \psi + \frac{c}{B_0} \left\langle (\hat{\mathbf{z}} \times \nabla \phi \cdot \nabla) \nabla_{\perp}^2 \phi \right\rangle = 0, \quad (10)$$

when  $\omega_{\rm pi} \gg \omega_{\rm ci}$ . For the Hasegawa-Mima azimuthally symmetric zonal flows, we insert (4) into the ion continuity equation and use (9) to obtain

$$\partial_t (1 - \rho_{\rm s}^2 \nabla_{\perp}^2) \psi + \rho_{\rm s}^2 \mu_{\rm i} \nabla_{\perp}^4 \psi - \frac{c}{B_0} \rho_{\rm s}^2 \left\langle (\hat{\mathbf{z}} \times \nabla \phi \cdot \nabla) \nabla_{\perp}^2 \phi \right\rangle = 0.$$
(11)

The last terms in the left-hand side of equations (10, 11) represent the ponderomotive force of coherent drift waves. They are associated with the nonlinear ion polarization drift.

Equations (7, 8, 10, 11) are the desired equations for studying the excitation of zonal flows by large amplitude drift waves.

# 3 Nonlinear dispersion relations and growth rates

The nonlinear interactions between a finite amplitude drift pump wave  $(\omega_0, \mathbf{k}_0)$  and zonal flows  $(\omega, \mathbf{k})$  excite upper and lower drift sidebands  $(\omega_{\pm}, \mathbf{k}_{\pm})$ . Thus, we decompose the drift wave potential as

$$\phi = \phi_{0+} \exp(-\mathrm{i}\omega_0 t + \mathrm{i}\mathbf{k}_0 \cdot \mathbf{r}) + \phi_{0-} \exp(\mathrm{i}\omega_0 t - \mathrm{i}\mathbf{k}_0 \cdot \mathbf{r}) + \sum_{+,-} \phi_{\pm} \exp(-\mathrm{i}\omega_{\pm} t + \mathrm{i}\mathbf{k}_{\pm} \cdot \mathbf{r}), \quad (12)$$

where  $\omega_{\pm} = \omega \pm \omega_0$  and  $\mathbf{k}_{\pm} = \mathbf{k} \pm \mathbf{k}_0$  are the frequencies and wave vectors of the sidebands, and the superscript 0 ( $\pm$ ) stands for the pump (sidebands).

Inserting (12) into (7) and (8) and Fourier analyzing we obtain

$$D_{\pm}\phi_{\pm} = \pm \mathrm{i}\frac{c}{B_0 a_{\pm}}\hat{\mathbf{z}} \times \mathbf{k} \cdot \mathbf{k}_0 (1 + k_{\perp 0}^2 \rho_{\mathrm{s}}^2)\phi_{0\pm}\hat{\psi},\qquad(13)$$

and

$$D_{\pm}\phi_{\pm} = \pm \mathrm{i} \frac{c}{B_0 a_{\pm}} \hat{\mathbf{z}} \times \mathbf{k} \cdot \mathbf{k}_0 q_{\perp}^2 \rho_{\mathrm{s}}^2 \phi_{0\pm} \hat{\psi}, \qquad (14)$$

where  $D_{\pm} = \omega_{\pm} - \omega_{*\pm}$ ,  $\omega_{*\pm} = k_{y\pm}U_*/a_{\pm}$ ,  $a_{\pm} = 1 + k_{\perp\pm}^2 \rho_s^2$ , and  $q_{\perp}^2 = k_{\perp0}^2 - k_{\perp}^2$ . In deriving equations (13, 14) we have introduced  $\psi = \hat{\psi} \exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r})$  and matched the phasors.

Furthermore inserting (12) into (10) and (11) and Fourier analyzing, we have, respectively,

$$(\omega + \mathrm{i}\Gamma_z)\hat{\psi} = \mathrm{i}\frac{c}{B_0}\frac{\hat{\mathbf{z}} \times \mathbf{k}_0 \cdot \mathbf{k}}{k_\perp^2} \times \left(K_-^2\phi_{0+}\phi_- - K_+^2\phi_{0-}\phi_+\right), \quad (15)$$

and

$$\left( \omega + i \frac{k_{\perp}^2 \rho_s^2 \Gamma_z}{1 + k_{\perp}^2 \rho_s^2} \right) \hat{\psi} = i \frac{c}{B_0} \frac{\hat{z} \times \mathbf{k}_0 \cdot \mathbf{k}}{1 + k_{\perp}^2 \rho_s^2} \rho_s^2 \\ \times \left( K_-^2 \phi_{0+} \phi_- - K_+^2 \phi_{0-} \phi_+ \right),$$
(16)

where  $K_{\pm}^2 = k_{\perp\pm}^2 - k_0^2$  and  $\Gamma_z = \mu_i k_{\perp}^2$ .

Combining equations (13–16) we readily obtain the nonlinear dispersion relations

$$\omega + i\Gamma_{z} = -\frac{c^{2}|\phi_{0}|^{2}}{B_{0}^{2}} \frac{|\hat{\mathbf{z}} \times \mathbf{k}_{0} \cdot \mathbf{k}|^{2}}{k_{\perp}^{2}} \times (1 + k_{\perp 0}^{2}\rho_{s}^{2}) \sum_{+,-} \frac{K_{\pm}^{2}}{a_{\pm}D_{\pm}}, \qquad (17)$$

and

$$\omega + i \frac{k_{\perp}^2 \rho_s^2 \Gamma_z}{1 + k_{\perp}^2 \rho_s^2} = -\frac{c^2 |\phi_0|^2}{B_0^2} \frac{|\hat{\mathbf{z}} \times \mathbf{k}_0 \cdot \mathbf{k}|^2}{1 + k_{\perp}^2 \rho_s^2} \times q_{\perp}^2 \rho_s^4 \sum_{+,-} \frac{K_{\pm}^2}{a_{\pm} D_{\pm}}, \qquad (18)$$

where  $|\phi_0|^2 = \phi_{0+}\phi_{0-}$ . For  $|\omega| \gg \Gamma_z$  and  $k_{\perp 0} \gg k_{\perp}$  we can express (17) and (18) as

$$\omega^2 \approx -2 \frac{c^2 |\phi_0|^2}{B_0^2} \frac{|\hat{\mathbf{z}} \times \mathbf{k}_0 \cdot \mathbf{k}|^2}{k_\perp^2} \mathbf{k}_0 \cdot \mathbf{k}_\perp, \qquad (19)$$

and

$$\omega^{2} \approx -2 \frac{c^{2} |\phi_{0}|^{2}}{B_{0}^{2}} \frac{|\hat{\mathbf{z}} \times \mathbf{k}_{0} \cdot \mathbf{k}|^{2}}{1 + k_{\perp}^{2} \rho_{\rm s}^{2}} \frac{\mathbf{k}_{0} \cdot \mathbf{k}_{\perp} k_{\perp 0}^{2} \rho_{\rm s}^{4}}{1 + k_{\perp 0}^{2} \rho_{\rm s}^{2}}, \qquad (20)$$

which can depict a purely growing  $(\omega = i\gamma_z)$  instability. For  $\mathbf{k}_0 \cdot \mathbf{k}_{\perp} > 0$  the increment for the azimuthally symmetric zonal flow excitation is

$$\gamma_z = \sqrt{2} \left| \frac{c\phi_0}{B_0} \frac{\hat{\mathbf{z}} \times \mathbf{k}_0 \cdot \mathbf{k}}{k_\perp} \right| \left| \mathbf{k}_0 \cdot \mathbf{k}_\perp \right|^{1/2}.$$
 (21)

The expression (21) predicts that the growth rate of the purely growing mode is directly proportional to the pump wave electric field  $\mathbf{k}_0 |\phi_0|$ . On the other hand, equation (20) reveals that the growth rate for the Hasegawa-Mima zonal flows is smaller than that for the azimuthally symmetric zonal flow excitation. Let us finally estimate the growth time for a typical laboratory plasma [15,16] with  $B_0 \sim 3$  kG,  $T_e \sim 50-100$  eV,  $\rho_s = 0.5-1$  cm, and  $\phi_0 \sim 1$  V. It turns out that zonal flows with a perpendicular scale size of a cm would have an e-folding time of five microseconds when the pump wavelength is of the order of ten cm (corresponding to  $k_{\perp 0}\rho_s \sim 0.5$ ). Thus, zonal flows can be parametrically excited on account of the free energy stored in the drift pump wave.

### 4 Nonlinear zonal flows

Parametrically excited zonal flows attain large amplitudes and then interact among themselves. That self-interaction between finite amplitude potential perturbations of zonal flows is the dominant nonlinearity, and it appears in the nonlinear ion polarization drift. The self-organization of fully developed zonal flow turbulence in the absence of the drift mode driver is then governed by the Navier-Stokes equation

$$D_t \nabla_\perp^2 \psi = 0 \tag{22}$$

for the azimuthally symmetric zonal flows, and by the Hasegawa-Mima equation

$$D_t (1 - \rho_{\rm s}^2 \nabla_\perp^2) \psi = 0 \tag{23}$$

for the zonal flows with  $n_{e1} = n_0 e\psi/T_e$ . Here we have used the notation  $D_t = \partial_t + (c/B_0)\hat{\mathbf{z}} \times \nabla \psi \cdot \nabla$ . Possible stationary solutions of (22) and (23) may appear in the form of monopolar vortices as well as counter-rotating vortices. As an illustration, we note that in the steady state, equations (22, 23) are satisfied by

$$\nabla^2_{\perp}\psi = G(\psi), \tag{24}$$

where  $G(\psi)$  is an arbitrary function. Choosing

$$G(\psi) = -U_0 \sinh\left(\frac{\psi}{\psi_c}\right), \qquad (25)$$

where  $U_0$  and  $\psi_c$  are arbitrary constants, one finds that the solution of the sinh-Poisson equation (24) is of the form [18,19]

$$\psi = -4\psi_{\rm c} \operatorname{arctanh}\left[\frac{\epsilon \cos(y)}{\cosh(\epsilon x)}\right],$$
(26)

which represents a street of counter-rotating Mallier-Maslowe vortices [18], where  $\epsilon$  (< 1) is a constant. Furthermore, choosing

$$G(\psi) = \frac{4\alpha K^2}{\gamma^2} \exp\left(-\frac{2}{\alpha}\psi\right),\qquad(27)$$

where  $\alpha$ , K and  $\gamma$  are constants, one finds that the solution of the Liouville equation (24) is of the form [19]

$$\psi = \alpha \ln \left[ 2\cosh(Kx) + 2\left(1 - \frac{1}{\gamma^2}\right)^{1/2} \cos(Ky) \right], \quad (28)$$

which is an even (in the x-direction) row of identical vortices for  $\gamma^2 > 1$ .

On the other hand, it should be stressed that the interaction between nonlinear zonal flows and the drift pump wave may significantly influence the pump wave as well. In such a situation, one can either use a model similar to that in reference [20] or carry out a numerical analysis of the relevant governing equations to investigate the interplay between drift waves and nonlinear zonal flows.

### 5 Summary

In summary, we have shown that finite amplitude drift waves can parametrically excite zonal flows in a nonuniform magnetoplasma. It has been found that azimuthally symmetric zonal flows with insignificant density fluctuations are driven faster than those zonal flows which have  $n_{e1} \sim n_0 e\psi/T_e$ . For typical laboratory parameters [15, 16], the growth time is of the order of five microseconds. Finite amplitude zonal flows undergo further nonlinear evolution leading to the formation of coherent vortex structures that are governed by the Navier-Stokes and the Hasegawa-Mima equations. Such coherent structures constitute a dynamical paradigm for intermittency in a nonuniform magnetoplasma containing nonlinearly coupled drift wavezonal flow turbulence. Hence, the present investigation provides a better understanding of drift wave induced sheared (zonal) flows which are observed in computer simulations [14] as well as in laboratory experiments [15,16]. Nonlinear sheared flows, in turn, would have an impact on the confinement of charged particles and transport barriers in tokamaks [21].

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